Geometric Singular Perturbation Theory for Electrical Circuits(Theory of Dynamical Systems and Its Application to Nonlinear Problems)

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§1. Introduction

An electrical circuit $\tilde{\mathcal{N}}$ is called a network perturbation of an electrical circuit $\mathcal{N}$ if $\tilde{\mathcal{N}}$ is obtained by adding to $\mathcal{N}$ some parasitic elements in a canonical way. T. Matsumoto asked to the author how can we treat the dynamics of a network perturbation $\tilde{\mathcal{N}}$ in relation to the dynamics of $\mathcal{N}$. The difficulty exist in the fact that the configuration manifolds $\tilde{\Sigma}$ of $\tilde{\mathcal{N}}$ and Σ of $\mathcal{N}$, on which the dynamical systems are defined, are placed in different spaces. Owing to the propositions in Section 2, we can conquer this difficulty.

By using these propositions we show two theorems. Theorem 3.3 is related to network R-perturbations. In these perturbations parasitic elements consist of resistors. Theorem 4.4 is related to network LC-perturbations. In this situation the perturbation $\tilde{\mathcal{N}}$ is obtained by adding to $\mathcal{N}$ parasitic inductors and capacitors. It is well known that the dynamical system of a network LC-perturbation is reduced to the singular perturbation theory of constrained differential equations. ([1],[2],[7],[9],[10],[11],[12]). But the situation seems to have been restricted in a local chart of the configuration manifold $\tilde{\Sigma}$ (i.e. $\tilde{\Sigma}$ have been assumed to be an Euclidean space) in a fibre bundle [12]. The aim of Theorem 4.4 is the reduction of the situation to global theories.

The main part of this paper consists of a reformulation of announcement [5] and the proof of Theorem 3.3.

§2. Fundamental propositions.

Let G be the oriented and connected graph of the given electrical circuit $\mathcal{N}$ with resistors, inductors and capacitors. Let $C_k(G)$ and $C^k(G)$ be the real k-chain complex and a real k-cochain complex of G, $k=0,1$. A state of the circuit $\mathcal{N}$ is specified by a current vector $i=(i_1,\cdots,i_b)\in C_1(G)$ and a voltage vector $v=(v_1,\cdots,v_b)\in C^1(G)$, where b is the number of the branches contained in G. Sometimes, we will use the notations $C_1(\mathcal{N})$ and $C^1(\mathcal{N})$ in the meaning of $C_1(G)$ and $C^1(G)$.

The Kirchhoff's law restricts the possible states of $\mathcal{N}$ to a b-dimensional linear subspace of $C_1(G)\times C^1(G)$ called the Kirchhoff space $K=\operatorname{Ker}\partial\times\operatorname{Im}\partial^*$, where ∂ is the boundary operator $\partial: C_1(G)\to C_0(G)$ and ∂^* is the coboundary operator $\partial^*: C^0(G)\to C^1(G)$.

<u>2.1 Definition</u>. A graph $\tilde{G}$ is a <u>fundamental</u> <u>extension</u> of G if $\tilde{G}$ is obtained by adding branches to G in the following way.

(S) Insert finite (may be zero) extra branches in series with each branches of G.

(P) Connect each pair of vertices of G by finite (may be zero) number of parallel extra branches.

Let T be a maximal tree of G. Then $L=G\setminus T$ is the link of T. We can take a maximal tree $\tilde{T}$ of $\tilde{G}$ such that

(i) $T \subset \tilde{T}$, and (ii) every extra branch inserted by (S) is contained in $\tilde{T}$. Consequently, every extra branch γ_j added by (P) is contained in the link $\tilde{L}$ of $\tilde{T}$. In fact, the vertices a, b in the boundary of γ is contained in T. So that a, b are contained in $\tilde{T}$. Hence γ_j cannot contained in $\tilde{T}$. Therefore, we have

$$G = T \cup L, \quad \tilde{G} = \tilde{T} \cup \tilde{L} \qquad (2.1)$$
$$T \subset \tilde{T}, \quad L \subset \tilde{L}.$$

Let $T' = \tilde{T} \setminus T$ and $L' = \tilde{L} \setminus L$. Then, T' and L' are the graphs consisting of the extra branches added by (S) and (P) respectively.

Let $\tilde{\partial} : C_1(\tilde{G}) \to C_0(\tilde{G})$ and $\tilde{\partial}^* : C^0(\tilde{G}) \to C^1(\tilde{G})$ be the boundary and coboundary operators. Kirchhoff space of $\tilde{G}$ is given by $\tilde{K} = \mathrm{Ker}\,\tilde{\partial} \times \mathrm{Im}\,\tilde{\partial}^*$. Since $\tilde{G} = G \cup T' \cup L'$, the elements $\tilde{i} \in C_1(\tilde{G})$ and $\tilde{v} \in C^1(\tilde{G})$ are decomposed as

$$\tilde{i} = (\tilde{i}_G, \tilde{i}_{T'}, \tilde{i}_{L'}) \quad \text{and} \quad \tilde{v} = (\tilde{v}_G, \tilde{v}_{T'}, \tilde{v}_{T'}). \qquad (2.2)$$

Put

$$(\mathrm{Ker}\,\tilde{\partial})_0 = \{ \tilde{i} \in \mathrm{Ker}\,\tilde{\partial} \,;\, \tilde{i}_{L'} = 0 \}$$
$$(\mathrm{Im}\,\tilde{\partial}^*)^0 = \{ \tilde{v} \in \mathrm{Im}\,\tilde{\partial}^* : \tilde{v}_{T'} = 0 \} \text{ and} \qquad (2.3)$$
$$K_0 = (\mathrm{Ker}\,\tilde{\partial})_0 \times (\mathrm{Im}\,\tilde{\partial}^*)^0.$$

Then, K_0 is a linear subspace of $\tilde{K}$.

Define projections

$$\pi_0 : C_1(\tilde{G}) \to C_1(G) \text{ and}$$
$$\pi^0 : C^1(\tilde{G}) \to C^1(G) \qquad (2.4)$$

by $\quad \pi_0(\tilde{i}) = \tilde{i}_G \quad$ and $\quad \pi^0(\tilde{v}) = \tilde{v}_G$.

2.2 Proposition. (i) π_0 maps $(\mathrm{Ker}\,\tilde{\partial})_0$ isomorphically onto $\mathrm{Ker}\,\partial$. (ii) π^0 maps $(\mathrm{Im}\,\tilde{\partial}^*)^0$ isomorphically onto $\mathrm{Im}\,\partial^*$. So that $(\pi_0 \times \pi^0)\,|\,K_0 : K_0 \to K$ is an isomorphism.

Let r, ℓ and c be the number of the resistors, inductors and capacitors in circuit $\mathcal{N}$. Let $\mathcal{R}$, $\mathcal{L}$ and $\mathcal{C}$ be the subgraph of G which consists of the branches that correspond to the resistors, inductors and capacitors of $\mathcal{N}$, respectively. Now we have $b = r + \ell + c$.

The following are the standing hypotheses of this paper:

2.3 Assumption.

(A) The graph of a circuit is connected.

(B) A circuit is time-invariant.

(C) The resistor constitutive relations are characterized by $(i_R, v_R) \in \Lambda_R$,

where Λ_R is a r-dimensional submanifold of class C^s, $s \geqq 2$, in $R^{2r} \cong C_1(R) \times C^1(R)$. Λ_R is called the characteristic manifold of $\mathcal{R}$.

In addition, we will set hypotheses about inductors and capacitors in the next section. When we consider a dyanmical system of a circuit, Λ_R will be assumed to be a manifold of class C^r, $r \geqq 2$.

We have the following natural direct sum decompositions.

$$C_1(\mathcal{N}) = C_1(\mathcal{R}) + C_1(\mathcal{L}) + C_1(\mathcal{C}) \cong \mathbb{R}^r \times \mathbb{R}^\ell \times \mathbb{R}^c$$

$$C^1(\mathcal{N}) = C^1(\mathcal{R}) + C^1(\mathcal{L}) + C^1(\mathcal{C}) \cong \mathbb{R}^r \times \mathbb{R}^\ell \times \mathbb{R}^c$$

By the hypothesis (C) above, the state $(i, v) \in C_1(\mathcal{N}) \times C^1(\mathcal{N})$ of $\mathcal{N}$ is contained in a submanifold

$$\Lambda = \Lambda_R \times C_1(\mathcal{L} \cup \mathcal{C}) \times C^1(\mathcal{L} \cup \mathcal{C}) \approx \Lambda_R \times \mathbb{R}^{2(\ell+c)}.$$

Here, the $\approx$ denotes to be diffeomorphic. Let $\mathcal{C}^r(\Lambda_R, \mathbb{R}^{2r})$ be the space of the C^r-mappings of Λ_R into $\mathbb{R}^{2r}$ with Whitney C^r-topology, where we consider $\mathbb{R}^{2r}$ as $C_1(\mathcal{R}) \times C^1(\mathcal{R})$. Then, generically in the mapping space $\mathcal{C}^r(\Lambda_R, \mathbb{R}^{2r})$, the manifold $\Lambda = \Lambda_R \times R^{2(\ell+c)}$ and Kirchhoff space K^b intersect transversely in $\mathbb{R}^{2b} = C_1(\mathcal{N}) \times C^1(\mathcal{N})$. This is shown in Theorem 5 [8] in class C^1 case, but the extension to C^r-version is trivial. Hence, we may assume that $\Sigma = \Lambda \cap K$ is a $(\ell + c)$-dimensional C^r manifold. We say that Σ is the configuration manifold of the circuit $\mathcal{N}$. Λ_R is called a characteristic manifold.

2.4 Proposition. Let $\tilde{G}$ be a fundamental extension of G. Let Λ be a C^r submanifold in $C_1(G) \times C^1(G)$ having transverse intersection with the Kirchhoff space K. Put $\tilde{\Lambda} \equiv \Lambda \times C_1(L' \cup T') \times C^1(L' \cup T') \subset C_1(\tilde{G}) \times C^1(\tilde{G})$, $\tilde{\Sigma} \equiv \tilde{\Lambda} \cap \tilde{K}$ and $\Sigma_0 \equiv \tilde{\Sigma} \cap K_0$. Then (i) the following intersections are transverse; $\tilde{\Lambda} \pitchfork \tilde{K}$ in $C_1(\tilde{G}) \times C^1(\tilde{G})$ and $\tilde{\Sigma} \pitchfork K_0$ in $\tilde{K}$ so that $\tilde{\Sigma}$ and Σ_0 are C^r manifolds, and (ii) $\pi_0 \times \pi^0$ maps Σ_0 diffeomorphically onto Σ.

§3. Network R-perturbation.

3.1 Definiton. A circuit $\tilde{\mathcal{N}}$ is a network R-perturbation of $\mathcal{N}$ if $\tilde{\mathcal{N}}$ is obtained by adding branches to $\mathcal{N}$ by following ways:

(S) To every branch of $\mathcal{N}$ insert in series an extra branch of resistor such that the set of these inserted resistors has "small resistance" (may be zero).

(P) Connect each pair of vertices of $\mathcal{N}$ by a branch of resistor such that the set of these resistors has "large resistance" (may be ∞).

Here, the above "small resistance" and "large resistance" are in the following meanings. Let $\mathcal{R}_0$ and $\mathcal{R}_\infty$ be the set of the branches of the resistors which are added to $\mathcal{N}$ by (S) and (P) above, respectively. Then "small resistance" means that the characteristic C^r-manifold $\Lambda_0 \subset C_1(\mathcal{R}_0) \times C^1(\mathcal{R}_0)$ is C^r close to the subspace $C_1(\mathcal{R}_0) \times \{0\} \subset C_1(\mathcal{R}_0) \times C^1(\mathcal{R}_0)$ in the space $C^r(C_1(\mathcal{R}_0), C_1(\mathcal{R}_0) \times C^1(\mathcal{R}_0))$ of all C^r mappings, $C_1(\mathcal{R}_0) \to C_1(\mathcal{R}_0) \times C^1(\mathcal{R}_0)$, with Whitney C^r topology. "Large resistance" means that $\Lambda_\infty \subset C_1(\mathcal{R}_\infty) \times C^1(\mathcal{R}_\infty)$ is C^r close to $\{0\} \times C^1(\mathcal{R}_\infty) \subset C_1(\mathcal{R}_\infty) \times C^1(\mathcal{R}_\infty)$. If they are "sufficiently close", we say that $\tilde{\mathcal{N}}$ is a "<u>sufficiently small</u>" network R-perturbation of $\mathcal{N}$.

<u>3.2. Lemma. Let K be a linear subspace and Λ be a C^r submanifold of an Euclidean space $\mathbb{R}^n$ such that they have transverse intersection. Let $F : \Lambda \to \mathbb{R}^n$ be an embedding such that it is a C^r perturbation of the identity mapping.</u>

<u>Then, there is a C^r embedding $G : \Sigma = \Lambda \cap K \to K$ such that $G(\Sigma) = F(\Lambda) \cap K$. Especially, G can be taken arbitrarily close to the identity mapping, if F is sufficiently close to the identity mapping.</u>

If we replace the differentiability of G_ε by C^{r-1}, then this lemma becomes well known. When $r=1$, this is Proposition 1 of [8].

Let $\tilde{\mathcal{N}}$ be a network R-perturbation of $\mathcal{N}$ and $\tilde{\Lambda}$ be the characteristic manifold, i.e. $\tilde{\Lambda} = \Lambda \times \Lambda_0 \times \Lambda_\infty$. Let $\mathcal{L}$ and $\mathcal{C}$ be the sets of inductor-branches and capacitor-branches, resp. $\tilde{\Sigma} = (\tilde{\Lambda} \times C_1(\mathcal{L} \cup \mathcal{C}) \times C^1(\mathcal{L} \cup \mathcal{C})) \cap \tilde{K}$ is the configuration manifold of $\tilde{\mathcal{N}}$, where $\tilde{K}$ is the Kirchhoff space of $\tilde{\mathcal{N}}$. Denote $\bar{\Lambda} = (\Lambda \times (C_1(\mathcal{R}_0) \times 0^*) \times (C^1(\mathcal{R}_\infty) \times 0_*))$, where $0^* \in C^1(\mathcal{R}_0)$ and $0_* \in C_1(\mathcal{R}_\infty)$ are zero vectors. Put $\bar{\Sigma} = (\bar{\Lambda} \times C_1(\mathcal{L} \cup \mathcal{C}) \times C^1(\mathcal{L} \cup \mathcal{C})) \cap \tilde{K}$. By Lemma 3.2, there is a diffeomorphisms $f : \bar{\Sigma} \to \tilde{\Sigma}$ which is arbitrarily close to the inclusion mapping $\bar{\Sigma} \to \tilde{K}$ if $\tilde{\Lambda}$ is sufficiently close to $\bar{\Lambda}$.

Attention to the equality,

$$\bar{\Sigma} = \{(\Lambda \times C_1(\mathcal{L} \cup \mathcal{C}) \times C^1(\mathcal{L} \cup \mathcal{C})) \times (C_1(\mathcal{R}_0 \cup \mathcal{R}^\infty) \times C^1(\mathcal{R}_0 \cup \mathcal{R}^\infty))\} \cap K_0,$$

where $K_0 = \{(i, v) \in \tilde{K} ; i_{R_\infty} = 0, v_{\mathcal{R}_0} = 0\}$. Since the graph of $\tilde{\mathcal{N}}$ is a fundamental extension of that of $\mathcal{N}$, by Proposition 2.4, we have $\pi_0 \times \pi^0$ maps $\bar{\Sigma}$ diffeomorphically onto the configuration manifold Σ of $\mathcal{N}$, where $\pi_0 : C_1(\tilde{\mathcal{N}}) \to C_1(\mathcal{N})$ and $\pi^0 : C^1(\tilde{\mathcal{N}}) \to C^1(\mathcal{N})$ are the projections defined by (2.4).

Let the C^{r-1} non-zero real functions $L_j(i_j)$ and $C_k(v_k)$ be the inductance and capacitance of the inductor L_j and the capacitor C_k of the circuit $\mathcal{N}$, respectively. The dynamcial system of $\mathcal{N}$ is the C^{r-1} vector field on the configuration C^r manifold Σ of $\mathcal{N}$, which is defined by the following equations;

$$v_j = L_j(i_j)\frac{di_j}{dt}, \quad j = 1, 2, \cdots, \ell. \tag{3.1}$$

$$i_k = C_k(v_k)\frac{dv_k}{dt}, \quad k = 1, 2, \cdots, c. \tag{3.2}$$

Here, i_j and v_j denote the current and voltage, respectively, of the inductor branch L_j, and i_k, v_k similarly of the capacitor branch C_k. $L_j(i_j)$ and $C_k(v_k)$ are C^{r-1} real valued functions, called as the inductance of L_j and the capacitance of C_k, respectively.

We have $\dim \Sigma = \ell + c$. A point x in Σ is said to be a <u>singular point</u>, if the differential

$$d(\pi|\Sigma)(x): T_x\Sigma \to C_1(\mathcal{L}) \times C^1(\mathcal{C})$$

does not have full rank. If otherewise, x is called a <u>regular point</u>. Here,

$$\pi : C_1(\mathcal{N}) \times C^1(\mathcal{N}) \to C_1(\mathcal{L}) \times C^1(\mathcal{C}) \tag{3.3}$$

is the natural projection, and $T_x\Sigma$ the tangent space of Σ at x. The set Σ_r of all regular point of Σ is called a <u>regular domain</u>, which is an open set of Σ. $\mathcal{N}$ is said to be <u>regular</u> if $\Sigma = \Sigma_r$. If $\mathcal{N}$ is regular, the vector of the dynamical system is defined at every point of Σ.

Let M and N be C^r manifolds and $f: M \to N$ be a C^r diffeomorphism. Let X be a vector field on M. Then a vector field $Y = f_*X$ on N is defined by

$$Y_{fx} \equiv (df)_x \cdot X_x$$

where X_x denotes the vector of X at x.

Let X be the dynamical system of $\mathcal{N}$, which is the vector field on Σ. Let $\bar{X}$ be the vector field on $\bar{\Sigma}$ defined by the equations (3.1) and (3.2). Then, we have

$$((\pi_0 \times \pi^0)|\bar{\Sigma})_* \bar{X} = X.$$

Hence, <u>we may identify</u> $\bar{X}$ <u>with</u> X. Let $\tilde{X}$ be the dynamical system of $\tilde{\mathcal{N}}$, which is a vector field on $\tilde{\Sigma}$. There is a diffeomorphism $f : \bar{\Sigma} \to \tilde{\Sigma}$ which is a perturbation of the inclusion mapping $f : \bar{\Sigma} \to \tilde{K}$. Therefore the vectors $\bar{X}_x$ and $\tilde{X}_{fx}$ are close.

Let $\mathfrak{X}^r(M)$ denote the space of all C^r vector field on a manifold M with Whitney C^r topology (cf. [8]). A vector field X is <u>structurally stable</u>, if there is an open set U of X in $\mathfrak{X}^r(M)$ such that for every $Y \in U$ there is a homeomorphism $h : M \to M$ mapping every orbit of X onto an orbit of Y with preserving the orientations of orbits.

$\tilde{X}$ and $f_*\tilde{X}$ has the same orbit structure, infact, f maps every orbit of $\tilde{X}$ onto an orbit of $f_*\tilde{X}$. If f is sufficiently close to the inclusion, then $f_*\tilde{X}$ is arbitrarily close to $\bar{X}$ in $\mathfrak{X}^{r-1}(\bar{\Sigma})$.

Therefore, we have proved the following theorem.

<u>3.3 Theorem</u>. <u>Let</u> $\mathcal{N}$ <u>be a regular electrical circuit which dynamical system</u> X <u>is structural stable</u>. <u>Then, the dynamical system</u> $\tilde{X}$ <u>of a sufficiently small</u> R-<u>perturbation</u> $\tilde{\mathcal{N}}$ <u>of</u> $\mathcal{N}$ <u>has the same orbit structure as</u> X, <u>that is there is a diffeomorphism</u> $\tilde{\Sigma} \to \Sigma$ <u>of configuration manifolds which maps every orbit of</u> $\tilde{X}$ <u>onto an orbit of</u> X <u>with preserving the orientations of orbits</u>.

§4. Network LC-perturbation.

<u>4.1 Definition</u>. A circuit $\tilde{\mathcal{N}}$ is a network LC-perturbation of $\mathcal{N}$, if $\tilde{\mathcal{N}}$ is obtained by adding branches to $\mathcal{N}$ by following ways:

(S) To every branch of $\mathcal{N}$ insert in series at most one extra branch of inductor with small non-zero inductance.

(P) Connect each pair of vertices of $\mathcal{N}$ by at most one extra branch of capacitor with non-zero small capacitance.

Here, small inductance means the function $L_j(i_j)$ in (3.1) which is C^{r-1} close to the constant zero map in Whitney C^{r-1} topology, and similarly for small capacitance.

<u>4.2 Definition</u>. A network LC-perturbation $\tilde{\mathcal{N}}$ of $\mathcal{N}$ is called a <u>regularized</u> <u>network</u> <u>perturbation</u>, if $\tilde{\mathcal{N}}$ is regular.

<u>4.3 Remark</u>. If every resistance of $\mathcal{N}$ is current controled or voltage controled, then there is a regularized network perturbation. This is verified using the method of [4].

To a regularized network perturbation $\tilde{\mathcal{N}}$, a family of networks $\{\tilde{\mathcal{N}}_\varepsilon\ ;\ 0 \leq \varepsilon < \varepsilon_0\}$ is defined as follows: For $\varepsilon = 0$, $\tilde{\mathcal{N}}_0 \equiv \mathcal{N}$. For $\varepsilon > 0$, the graph of $\tilde{\mathcal{N}}_\varepsilon$ is the same as $\tilde{\mathcal{N}}$. If $L'_1(i_1), \cdots, L'_\ell(i_\ell)$ are the inductances of parasitic inductors of $\tilde{\mathcal{N}}$, then $\tilde{\mathcal{N}}_\varepsilon$ has the parasitic inductors of inductances $\varepsilon L'_1(i_1), \cdots, \varepsilon L'_\ell(i_\ell)$ at the same branches of the graph, has the parasitic capacitors similarly, and has the same elements as $\tilde{\mathcal{N}}$ at every other branches.

The graph of $\tilde{\mathcal{N}}_\varepsilon$, $\varepsilon > 0$, is a fundamental extention of that of $\mathcal{N}$. $\tilde{\mathcal{N}}_\varepsilon$ has the same configuration manifold $\tilde{\Sigma}$ for every $\varepsilon > 0$. By Proposition 2.2 and Proposition 2.3, there is a submanifold Σ_0 in $\tilde{\Sigma}$ such that Σ_0 is mapped by $\pi_0 \times \pi^0$ diffeomorphically onto the configuration manifold Σ of $\mathcal{N}$, where π_0 and π^0 are defined similarly as 2.4.

Let $\tilde{X}_\varepsilon$ be the dynamical system of $\tilde{\mathcal{N}}_\varepsilon$, which is a vector field on $\tilde{\Sigma}$. Here, we describe $\tilde{X}_\varepsilon$ in a local chart of $\tilde{\Sigma}$. Let $\mathcal{L}$, $\mathcal{C}$ and $\mathcal{R}$ be the inductors, capacitors and resistors of $\mathcal{N} \subset \tilde{\mathcal{N}}$; $\mathcal{L}'$ and $\mathcal{C}'$ be the parasitic inductors and capacitors of $\tilde{\mathcal{N}}$. Since $\tilde{\mathcal{N}}$ is regular, then for a small neighborhood U of every point in $\tilde{\Sigma}$ the mapping $\pi : U \to C_1(\mathcal{L} \cup \mathcal{L}') \times C^1(\mathcal{C} \cup \mathcal{C}')$ is a diffeomorphism onto the image, where π is the projection defined similarly as (3.3). Hence, U is the graph of the mapping, so that U is canonically coordinated by $(i_L, i_{L'}, v_C, v_{C'})$.

$$\left.\begin{aligned}
v_L &= f_L(i_L, i_{L'}, v_C, v_{C'}), \\
v_{L'} &= f_{L'}(\quad \cdots \quad), \\
i_C &= f_C(\quad \cdots \quad), \\
i_{C'} &= f_{C'}(\quad \cdots \quad), \\
i_R &= f_R(\quad \cdots \quad), \\
v_R &= f_R^*(\quad \cdots \quad).
\end{aligned}\right\} \qquad (4.1)$$

Using this coordinates, $\tilde{X}_\varepsilon$ is expressed by the following equation.

$$\left.\begin{aligned}
L(i_L)\cdot \dot{i}_L &= f_L(i_L, i_{L'}, v_C, v_{C'}) \\
C(v_C)\cdot \dot{v}_C &= f_C(\quad \cdots \quad) \\
\varepsilon\cdot L'(i_{L'})\cdot \dot{i}_{L'} &= f_{L'}(\quad \cdots \quad) \\
\varepsilon\cdot C'(v_{C'})\cdot \dot{v}_{C'} &= f_{C'}(\quad \cdots \quad)
\end{aligned}\right\} \qquad (4.2)_\varepsilon$$

Putting $\varepsilon = 0$ we have

$$\left.\begin{aligned} L(i_L)\cdot \dot{i}_L &= f_L\,, & C(v_C)\cdot \dot{v}_C &= f_C \\ 0 &= f_{L'}\,, & 0 &= f_{C'} \end{aligned}\right\} \qquad (4.2)_0$$

Since $\Sigma_0 = \{(i, v) \in \tilde{\Sigma}\ ;\ v_{L'} = 0,\ i_{C'} = 0\}$, then (4.1) implies that $(4.2)_0$ is the equation of the dynamical system of $\mathcal{N}$ on $\Sigma_0 \cap U$.

Define a vector field Y_ε on $\tilde{\Sigma}$ by

$$Y_\varepsilon \equiv \begin{cases} \varepsilon\cdot\tilde{X}_\varepsilon\,, & \text{if } \varepsilon \neq 0 \\ \lim\limits_{\varepsilon\to 0} \varepsilon\cdot\tilde{X}_\varepsilon\,, & \text{if } \varepsilon = 0 \end{cases} \qquad (4.3).$$

Since Y_0 vanishes identically on Σ_0, the differential $dY_0(x) : T_x\tilde{\Sigma} \to T_x\tilde{\Sigma}$ induces a linear map

$$QY_0(x) \ :\ T_x\tilde{\Sigma} / T_x\Sigma_0 \longrightarrow T_x\tilde{\Sigma} / T_x\Sigma_0$$

on the quotient space. Let $(\Sigma_0)_R \subset \Sigma_0$ be the open subset where QY_0 is invertible. $(\Sigma_0)_R$ is called the <u>normally regular domain</u> of Σ_0. Let $(\Sigma_0)_S \subset \Sigma_0$ be the open subset where all the eigenvalue of QY_0 has negative real part. $(\Sigma_0)_S$ is called the <u>normally stable domain</u> of Σ_0. For each $x \in (\Sigma_0)_R$, $T_x\Sigma_0$ has a unique complement N_x which is invariant under $dY_0(x)$. In fact N_x is the subspace of $T_x\Sigma_0$ generated by $(i_{L'}, v_{C'})$ in the above canonical coordinate system. N_x is a realization of $T_x\tilde{\Sigma} / T_x\Sigma_0$. Let ν be the projection

$$\nu : T\tilde{\Sigma}\,|\,(\Sigma_0)_R = (T\Sigma_0\,|\,(\Sigma_0)_R) \oplus N \longrightarrow T\Sigma_0\,|\,(\Sigma_0)_R .$$

In $(\Sigma_0)_R$ the <u>reduced vector field</u> X_R is defined by

$$X_R(x) = \nu \frac{\partial}{\partial\varepsilon} Y_\varepsilon(x)_{\varepsilon=0}\,, \qquad (4.4)$$

where we regard $Y_\varepsilon(x)$ as a vector field on $\tilde{\Sigma} \times [0, \varepsilon_0)$.

We have the following theorem.

4.4 Theorem. Let $\{\tilde{\mathcal{N}}_\varepsilon ; 0 \leqq \varepsilon < \varepsilon_0\}$ be the family of networks associated with a regularized network perturbation $\tilde{\mathcal{N}}$ of a network $\mathcal{N}$. Let Σ and $\tilde{\Sigma}$ be the configuration manifolds of $\mathcal{N}$ and $\tilde{\mathcal{N}}_\varepsilon (\varepsilon \neq 0)$, respectively.

Then, there is a canonically defined submanifold $\Sigma_0 \subset \tilde{\Sigma}$, which is identified with Σ by the natural projection $\pi_0 \times \pi^0 : (i_{\tilde{N}}, v_{\tilde{N}}) \mapsto (i_N, v_N)$. Let $\tilde{X}_\varepsilon$ be the dynamical system of $\tilde{\mathcal{N}}_\varepsilon$ on $\tilde{\Sigma}$, X the dynamical system of $\mathcal{N}$ on Σ_0, and Y_ε the vector field (defined by (4.3)). Then we have,

(i) $(\pi_0 \times \pi^0)$-image of normally regular domain $(\Sigma_0)_R$ coincides with the regular domain Σ_r, i.e. $(\pi_0 \times \pi^0)(\Sigma_0)_R = \Sigma_r$,

(ii) $X_R = X$, where X_R is the reduced vector field (defined by (4.4)).

4.5 Remark. This thoerem enable us to reduce the situation near the normally stable domain to the main theorem of [N. Fenichel, 3]. By Fenichel's theorem, there is a unique invariant manifold D_X^ε of $\tilde{X}_\varepsilon$ near $(\Sigma_0)_S$, with $\tilde{X}_\varepsilon | D_S^\varepsilon$ close to $X|(\Sigma_0)_S$, and that in a neighborhood of $(\Sigma_0)_S$ the motion of $\tilde{X}_\varepsilon$ is asymptotic to D_S^ε.

4.6 Remark. By the forthcoming paper [6] the following fact holds under a generical assumption: Let S be the set of singular point of $\mathcal{N}$ in the boundary of the normally stable domain $(\Sigma_0)_S$. There is an open and dense subset $\Gamma \subset S$ such that, if a trajectory of X come to a point in Γ, then it jumps into a point in another component of $(\Sigma_0)_S$ along a unique orbit.

this jumping is in the sense of singular perturbation.

4.7 Remark. The part (i) of the Theorem means that, though there may be many regularized network perturbation of $\mathcal{N}$, the important domain $(\Sigma_0)_R$ is unique in this sense.

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